Characteristics of vehicular traffic flow at a roundabout

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We construct a stochastic cellular automata model for the description of vehicular traffic at a roundabout designed at the intersection of two perpendicular streets. The vehicular traffic is controlled by a self-organized scheme in which traffic lights are absent. This controlling method incorporates a yield-at-entry strategy for the approaching vehicles to the circulating traffic flow in the roundabout. Vehicular dynamics is simulated and the delay experienced by the traffic at each individual street is evaluated. We discuss the impact of the geometrical properties of the roundabout on the total delay. We compare our results with traffic-light signalization schemes, and obtain the critical traffic volume over which the intersection is optimally controlled through traffic-light signalization schemes.

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I. INTRODUCTION

Undoubtedly, traffic management is nowadays considered as one of the basic ingredients of modern societies and large sums are invested by governments in order to increase its efficiency. The rapidly growing volume of vehicular traffic flow, limitations on expanding the construction of new infrastructure, together with unfavourable delays suffered in congested traffic jams, are among the basic features which necessitate the search for new control, as well as optimization schemes, for vehicular traffic flow. Inevitably, this task would not be significantly fulfilled unless a comprehensive survey of vehicular dynamics, within a mathematical framework, is developed. This has motivated physicists to carry out extensive numerical, as well as analytical research, in the discipline of *traffic flow theory*. The statistical physicists contribution to the field has accelerated since 1990's when computers provided the possibility of simulating traffic flow through the discretization of space and time. Ever since, a vast number of results, both analytically and empirically, has emerged in traffic discipline [1–13]. Broadly speaking, the traffic flow theory can be categorized into two parts: *Highway traffic* and *city traffic*, and now there is vast literature in both domains. In this paper, we focus out attention on a particular aspect of city traffic [14–19], the so-called *roundabout*. We try to present a numerical investigation on controlling urban traffic via roundabouts. Traditionally, the conflicting flows in urban areas were controlled by signalized intersections. Ever since the installation of first traffic light in New York in 1914, the subject of urban traffic and its optimal control has been intensively explored. Nowadays, the traffic in most urban areas is controlled by signalized intersections. Modern roundabouts have quite recently come into play as alternatives to signalized intersections, which tend to control the traffic flow more optimally and in a safer manner. A roundabout is a form of intersection design and control which accommodates traffic flow in one direction around a central island, operating with yield control at the entry points, and giving priority to vehicles within the roundabout (circulation flow). Yield-at-entry is the most important operational element of a modern roundabout, but it is not the only one. Deflection of the vehicle path and entry flare are also important characteristics that distinguish the modern roundabout from the nonconforming traffic circle, which does not have these characteristics. It has always been a subject of argument whether to control an intersection under a signalized or nonsignalized scheme via roundabouts. Apparently in low-volume situations, nonsignalized methods seem to show better performance; whereas, in high-volume traffic, one has to apply traffic light signalization [20–24]. The basic question which arises is: Under what circumstances should one control an intersection by signalized traffic lights? To address this fundamental question, we try to explore and analyze some basic characteristics of a typical roundabout, such as flow and delay, in order to find a quantitative understanding. In what follows, we try to illustrate these fascinating aspects through computer simulations.

II. DESCRIPTION OF THE PROBLEM

We now turn to discussing the simulation of traffic at a roundabout. A roundabout is a form of intersection design and control which accommodates traffic flow in one direction around a central island and gives priority to vehicles within the roundabout (circulation flow). Let us first discuss the basic driving principles applied to roundabouts. In its most general form, a roundabout connects four incoming, as well as four outgoing flow directions. In principle, each incoming vehicle approaching the roundabout can exit from each of four out-going directions via making appropriate turning maneuvers around the central island of the roundabout. Figure 1 illustrates the situation.

The rules of the road give the movement priority to those vehicles which are circulating around the central island. The approaching vehicles should yield to the circulating traffic flow in the roundabout, and are allowed to enter the roundabout, provided that some cautionary criteria are satisfied. In this paper, we only investigate a simplified version in which all the streets, including the circulating lane around the central island, are assumed to be single lane. Let us explain the entrance regulations in some details. Each approaching vehicle to the entry points of the roundabout should decelerate and simultaneously look at the leftward quadrant of the

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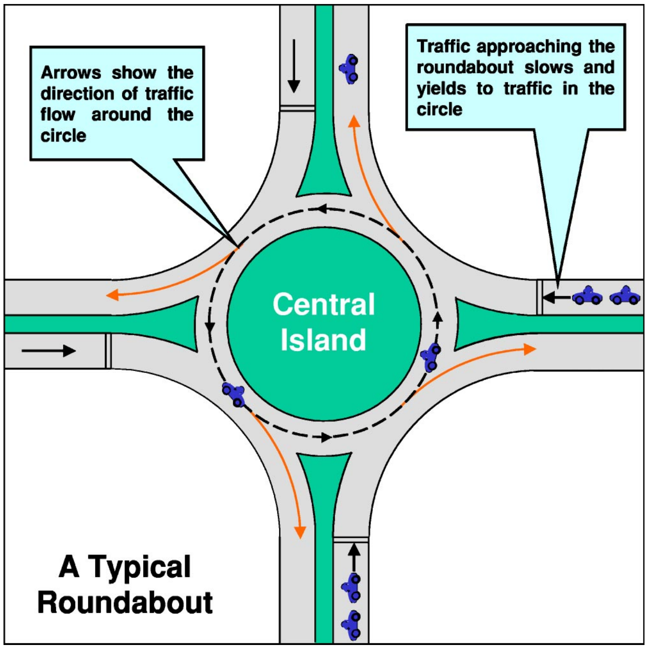

FIG. 1. A typical roundabout with yield-to-entry rules. Street A is south–north and street B is a west–east one.

roundabout. If there is a vehicle in this quadrant, then the approaching vehicle should come to a complete stop until the inside vehicle(s) leave(s) this quadrant. The stopped car is only allowed to enter the roundabout provided that no vehicle appears in its left side quadrant; otherwise, it has to slow down and stop. The stopped direction can flow as soon as the front car finds no car in the left-side quadrant of the circulating lane.

This is possible due to stochastic fluctuation in the space gap of the flowing direction. Once such an appropriate space gap has been found, the stopped car is allowed to enter the roundabout. This procedure is continuously applied to all approaching vehicles. Now we return to those vehicles which are moving around the interior island of the roundabout. Once a vehicle is permitted to enter the roundabout, it continues moving until it reaches to its aimed exit direction. Depending on the selected out-going direction, each interior vehicle moves a portion of the way around the central island. These turning movements are classified as: Right-turn, straight ahead, left-turn and U-turn. For those who tend to make a U-turn, the whole circumference should be traveled. The interior vehicle can freely move around the roundabout until it reaches the desired exit. The above-mentioned driving rules establish a mechanism responsible for controlling the traffic in conflicting points. This mechanism blocks any direction which is conflicting with a flowing one, thereby producing waiting queues in blocked directions. In contrast to signalized intersections, intersecting streets through roundabouts are controlled via a self-organized mechanism of blocking. It is evident that in congested traffic situations, where the in-flow rates are high, the probability of finding a large space gap is low. This leads to global blocking of other directions, which in turn gives rise to the formation of pronounced queues.

In this situation, the roundabout performance is inefficient, and apparently, a signalized control strategy shows a better performance. Conversely, in relatively low traffic volume it is likely to find a large space gap (by fluctuation) in the circulating direction and, hence, the possibility of entrance for the block direction increases. This increases the efficiency of the roundabout. The roundabout efficiency significantly depends on the incoming fluxes of cars and statistics of space gaps. The basic question raised is: *Under what circumstance the self-organized control scheme becomes inefficient?* In order to find better insight to the problem, we have simulated the traffic flow and have investigated the roundabout performance for various traffic situations and geometrical sizes of roundabout. In the subsequent sections, we present our simulation results.

III. FORMULATION OF THE MODEL

In this section, we begin with the simplest flow structure of the roundabout. In this case, the roundabout connects two single-lane one-way to one-way streets. With no loss of generality, we take the direction of flows to be northward on street A and eastward on street B (see Fig. 1). Also we give a permanent priority to the flow of street B, i.e., those cars which are driving on street B can enter the roundabout without any caution. On the contrary, the flow in street A should yield to the flow of street B. A-vehicles should observe the yield-at-entry rules. They are allowed to enter only by gap fluctuations in the B-flow. Correspondingly, no delay is wasted for B-vehicles. In order to capture the basic features of the problem, we have simulated the traffic movement in the framework of both *cellular automata* and *car-following* approaches. In what follows, we briefly explain the models rules and the numerical values of their parameters.

A. Modified Nagel-Schreckenberg cellular automata

In the cellular automata (CA) approach, space and time are discretized in such a way that each street is modeled by a one-dimensional chain divided into cells which are the same size as a typical car length. The circulating lane of the roundabout is also considered as a discretized closed chain. Time is assumed to elapse in discrete steps. We take the number of cells to be L for both streets and L_r for the interior lane. Each cell can be either occupied by a car or be empty. Moreover, each car can take discrete-valued velocities $1,2,\ldots,v_{\max}$. To be more specific, at each step of time, the system is characterized by the position and velocity configurations of the cars on each road. We note that due to the turning maneuvers, the maximum velocity of circulating cars should be reduced. Here we assume that the maximum velocity for interior cars takes the value of 40 km/h. The system evolves under a generalized discrete-time Nagel-Schreckenberg (NS) dynamics [25]. The generalized model incorporates the anticipation effects of driving habits [26]. It modifies the standard NS model at its second step, i.e., adjusting the velocities according to the space gap. We recall that due to the regular driving at the roundabouts, there is no need to use all aspects of the model introduced in [26], such as adaptation time-headway, brake-lights, etc. The implementation of the modified gap is sufficient for our purposes here. Let us briefly explain the updating rules which are synchronously applied to all the vehicles. We denote the position, velocity, and space gap (distance to its leading car) of a typical car at discrete time t by $x^{(t)}$, $v^{(t)}$, and $g^{(t)}$. The same quantities for its leading car are denoted by $x_l^{(t)}$, $v_l^{(t)}$, and $g_l^{(t)}$. Assuming that the expected velocity of the leading car, anticipated by the one following, in the next time step $t+1$ takes the form $v_{l,\text{anti}}^{(t)}=\min(g_l^{(t)},v_l^{(t)})$, we define the effective gap as $g_{\text{eff}}^{(t)}=g^{(t)}+\max(v_{l,\text{anti}}^{(t)}-\text{gap}_{\text{secure}},0)$ in which $\text{gap}_{\text{secure}}$ is the minimal security gap. Concerning the above-mentioned considerations, the following updating steps evolve the position and the velocity of each car.

(1) Acceleration: $v^{(t+1/3)}=\min(v^{(t)}+1,v_{\max})$,

(2) Velocity adjustment: $v^{(t+2/3)}=\min(g_{\text{eff}}^{(t+1/3)},v^{(t+1/3)})$,

(3) Random breaking with probability p: if random $<p$ then $v^{(t+1)}=\max(v^{(t+2/3)}-1,0)$, and

(4) Movement : $x^{(t+1)}=x^{(t)}+v^{(t+1)}$.

The state of the system at time $t+1$ is updated from that in time t by applying the modified NS dynamical rules. Let us now specify the physical value of our time and space units. The length of each cell is taken to be 5.6 m which is the typical bumper-to-bumper distance of cars in a waiting queue. Concerning the fact that in most of urban areas a speed-limit of 60 km/h should be kept by drivers, we quantify the time step in such a way that $v_{\max}=6$ corresponds to the speed-limit value (taken as 60 km/h). In this regard, each time step equals 2 s; and therefore, each discrete increment of velocity signifies a value of 10 km/h which is equivalent to a comfortable acceleration/deceleration of 1.4 m/s^2. We have also set the streets lengths as $L=70$ cells and $\text{gap}_{\text{sec}}=1$. We want to emphasize that the roundabout size, i.e., the circumference of the central island is a crucial parameter and should be carefully treated. At the end of each updating step, we evaluate the aggregate delay of street A. During the periods of the flow of B-vehicles in the interior lane, the A-vehicles are hindered, and accordingly, should stop before the interior island; hence, a queue will be formed. As soon as a car comes to a halt, it contributes to the total delay. In order to evaluate the delay, we measure the queue length (the number of stopped cars) at time step t, and denote it by the variable Q. Delay at time step $t+1$ is obtained by adding the queue length Q to the delay at time step t, i.e. $\text{delay}(t+1)=\text{delay}(t)+Q(t)$. This ensures that during the next time step, all of the stopped cars contribute one step of time to the delay. Let us now discuss the entrance of cars into the roundabout. So far, we have dealt with those cars within the horizon of the roundabout which goes up to the boundary points located at site L upstream from each incoming flow. It would be illustrative to discuss the entrance of cars into the roundabout. We take the distance of the boundary position to be 70 cells, equivalent to 400 m to the central island. The time headways between entering cars at this entry location vary in a random manner which consequently implies a random distance headway between successive entering cars. As a candidate for describing the statistical behavior of the random space gap of entering cars, we have chosen the Poisson distribution. The Poisson distribution function have been used in a variety of phenomena incorporating the modeling of "queue theories" [27]. According to this distribution function, the probability that the space gap between the car entering the intersection horizon and its predecessor be n is: $p(n)=\lambda^n e^{-\lambda}/n!$, where the parameter λ specifies the average as well as the variance of distribution function. The parameter $1/\lambda$ is proportional to the traffic volume. At the end of each time step, we evaluate the position of the most remote car to the roundabout border and denote it by x_{last}. We then draw a random integer number, n, from the above-mentioned Poisson distribution and create a newly entered car with $v_{\text{new}}=v_{\max}$ at $x_{\text{new}}=x_{\text{last}}+n$ provided the condition $x_{\text{new}}\leqslant L$ holds. Otherwise, the creation is rejected.

TABLE I. Model parameters and numerical values.

Parameter	Value	Unit
x_{neutral}	25	m
x_{width}	23.3	m
$v_{\max}$	17	m/s
α	0.1	s^{-}
c_{bias}	0.913	$\cdots$
Δt	0.1	s

B. Car-following approach: Optimal velocity

A substantially different approach in modeling the vehicular movement is based on the idea that each driver controls her speed under the stimuli received from the preceding car. These car-following models are mainly formulated within the framework of differential equations. Among various types of car-following models, *optimal-velocity* (OV) models have proven to successfully describe many realistic features of traffic flow [28,29]. Here, we have considered a coupled-map version of the OV model introduced in [30]. In this coupled-map OV model, the vehicular dynamics is governed by the following Newtonian-type differential equation:

$$\frac{dv}{dt}(t) = \alpha\{V_{\text{optimal}}[\Delta x(t)] - v(t)\}, \qquad (1)$$

where $x(t)$ and $v(t)$ denotes the position and the velocity of the vehicle at time t, respectively, and $\Delta x(t)$ is the space gap to the preceding car. The vehicle tries to keep a gap-dependent OV which is induced by the OV function. We adopt the same OV function as in [30]:

$$V_{\text{optimal}} = \frac{v_{\max}}{2}\left[\tanh\left(2\frac{\Delta x - x_{\text{neutral}}}{x_{\text{width}}}\right) + C_{\text{bias}}\right]. \qquad (2)$$

The model parameters have been calibrated to the empirical data [29]. Their numerical values are shown in Table I.

In Table I, Δt is the updating time interval for discretization of the dynamical equation. We have chosen Δt as 0.1 s in our simulation. Entrance of vehicles are analogous to the CA formulation. Before turning to the simulation results, it would be illustrative to discuss the nature of vehicular dynamics at the roundabout. Contrary to traffic flow at highways where interaction among vehicles gives rise to complex spatiotemporal traffic patterns, in the roundabout the traffic

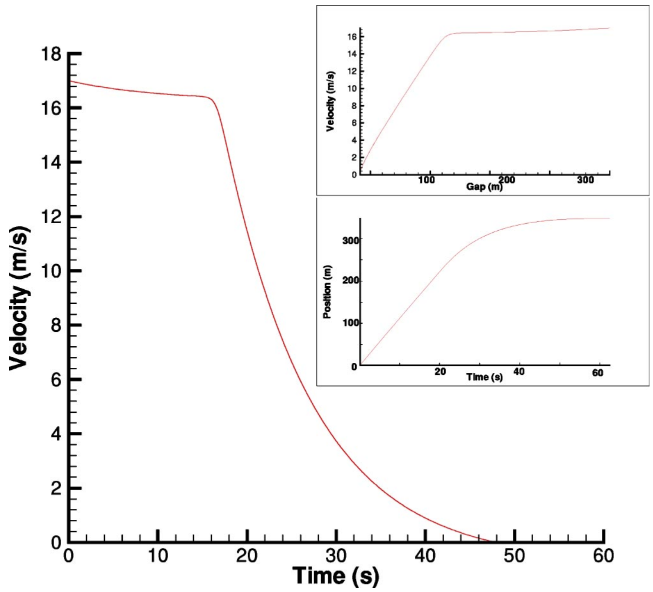

FIG. 2. Velocity of the approaching car to the queue. The lower inset graph shows the space–time diagram. The upper diagram shows the gap-velocity curve. α is chosen at 0.1.

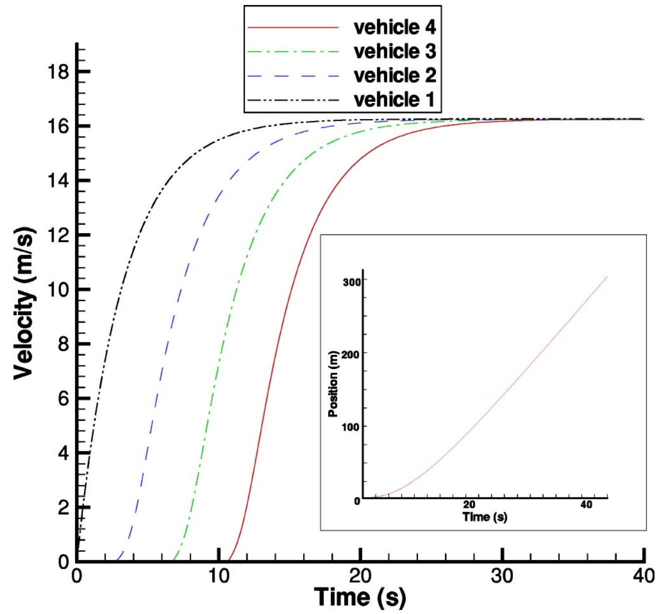

FIG. 3. Velocity-time plot of four standing cars after being permitted to move (by the fluctuation of flow in the perpendicular direction). α is set to 0.3. The inset shows the space-time trajectory of the pioneering vehicle.

flow is much more regulated. This regulation appears as the result of traffic rules at the roundabout. Yield-at-entry makes the vehicles slow down their speed in more or less the same manner. Our every day observation of traffic flow at roundabouts confirms this picture. Besides, the traffic volume should be relatively low for a roundabout to operate efficiently. As will be seen from our results, the efficient in-flow rates could be regarded, to a very good extent, as a free flow (we describe this point later in more details). Correspondingly simple models are sufficient to describe such an uncomplicated vehicular dynamics. We recall that when the congestion is above the free flow, the vehicular dynamic turns out to be so complicated that simple dynamics, such normal NS types or even optimal velocity types, fail to reproduce empirical observations. In those cases, one has to resort to realistic multiparameter models such as the three-phase cellular automata model introduced by Kerner, Klenov, and Wolf [31,32]. A successful modeling of roundabout should be able to correctly simulate the braking of cars upon reaching the queue. In Figs. 2 and 3, we depict the results of a single car braking upon approaching to a queue. We assume the car is driving at $v_{\max}=60$ km/h. Its initial distance to the queue is taken as 300 m.

As can be seen, the car drives at $v_{\max}$ up to a safe distance. After reaching this distance, it brakes. The braking acceleration is comfort and is of order 1 $\mathrm{m/s}^2$. Figure 3 shows the velocity-time diagrams of four vehicles stopped at a queue from the moment $(t=0)$ when the queue is allowed to move. The results are consistent with everyday experiences.

IV. SIMULATION RESULTS

We let the roundabout evolve for 1800 time steps which is equal to a real-time period of 1 h. We have averaged the results of 50 independent runs. Let us first consider the symmetric traffic states in which the traffic conditions are equal for both roads. In this case, we load the streets equally with approaching cars, spatially separated by a random space gap (Poisson statistics) from each other. Figure 4 depicts the total delay curves as a function of average space gap of entering cars $\lambda_A=\lambda_B=\lambda$ for various roundabout sizes. Vehicular dynamics is modified NS. All vehicles leave the roundabout along the incoming direction viz. they are not permitted to

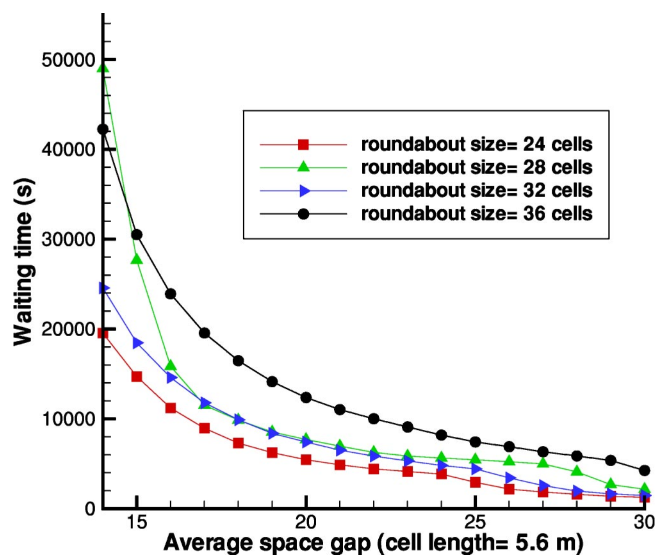

FIG. 4. Total delay vs the average space gap of approaching cars for various sizes of roundabout. The vehicular movement is evolved according to the modified NS dynamical rules.

turn right, left, or U-turn upon circulating the roundabout. According to the graph in Fig. 4, the delay shows a rapid decline for light traffic states. This marks the high efficiency of roundabout in low-volume traffic situations. In the heavy traffic situation, corresponding to short λ, we see the rapid increase of the delay. This is due to gridlocking in street A which appears as a result of scarcity in space gap of B-vehicles. Roundabouts are designed in different sizes to serve various objectives and conditions. Even miniroundabouts are effective at reducing speed and improving safety. Our simulation results confirm that roundabout size plays a dominant role in its performance. Figure 4 suggests the short-sized roundabouts operate more optimally. The reason is that for a short-sized roundabout, it would be easier for a waiting A-vehicle to find an empty quadrant than the one in a large-sized roundabout. This is due to fact the quadrant length is shorter in small roundabouts and that statistics of the headways in B-flow does not depend on the roundabout size.

A. Right-turn permission

At this stage, we remove parts of the restriction on the exit direction and enable each car to leave the roundabout at its first exit i.e., a right-turn. This implies that the south-north direction (street A) is equipped with an extra south-bound lane along which the incoming B-vehicle can leave the roundabout through a right-turn. Analogously, the approaching A-vehicles can leave the roundabout through the exit leg of street B via a right-turn maneuver. Therefore, for each incoming vehicle, we assign a parameter which determines the vehicle's decision to exit along the incoming direction or leave the roundabout at its first exit by making a right-turn maneuver. We denote this right-turn probability by σ_A and σ_B for incoming A- and B-vehicles, respectively. Before proceeding further, it would be illustrative to discuss the effect of displaying indicators. By the usage of indicators, each approaching vehicle can inform the others of his exit direction. Displaying the right-indicator corresponds to the case in which the driver intends to make a right-turn and leave the roundabout at the first exit. Those drivers who intend to exit straight ahead should not display their indicators. Indicator usage gives rise to an easier entrance to the roundabout and it may seems that the usage of an indicator decreases the delays. Our simulation results, nevertheless, prove the contrary. Figure 5 shows the waiting time for the symmetric situation in which the turning probability is taken equally as 1/2 for both A and B vehicles. Roundabout size is taken as 24 cells.

In spite of a more convenient entrance to the roundabout by displaying indicators, Fig. 5 depicts that the impact of displaying an indicator increases the overall delay. This result can be explained by the fact that although displaying indicators make cars enter the central island more conveniently, this leads to an increase of car density in the central island. Since the circulating vehicles should move slower for security reasons, this slows down the flow of B-vehicles which intend to go straightforward. This slowing down affects the headway distribution of B-vehicles. It actually makes the headways shorter and this intensifies the blocking of A-flow and hence increases the delay.

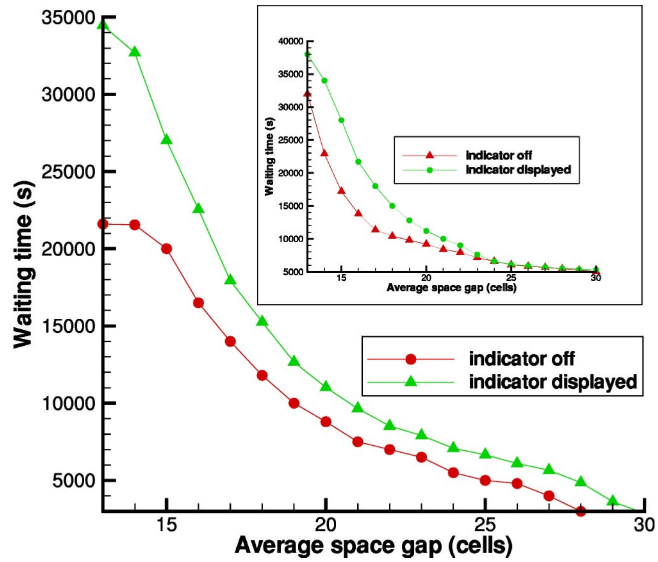

FIG. 5. Delay vs average space gap of approaching cars (symmetric in-flows) for two cases of indicators displayed and off (NS dynamics). The right-turn probability is equal to 0.5 for approaching cars of both streets. The inset sketches the above conditions in OV dynamics.

B. Left- and U-turn around the island

Let us now consider a more realistic situation. In its most general form, vehicles can enter from four directions, i.e., north, south, east, and west, to a roundabout. We denote these entries by S_{in}, N_{in}, W_{in}, and E_{in}, respectively. Moreover, there are four exit directions denoted by S_{out}, N_{out}, W_{out}, and E_{out}. Entering vehicles can exit from any of the outgoing directions by making an appropriate turning maneuver around the central island. Let us assume that vehicles enter only from S_{in} and W_{in} but can exit from every outgoing directions upon their decision.

In this case, B-vehicles should also yield to traffic in the roundabout since those vehicles intending to exit from the S_{out} have priority with respect to incoming cars from W_{in} entry, i.e., B-vehicles. Consequently, in this general case, both B- and A-vehicles contribute to delay. Figure 6 exhibits the overall delay for a 1 h performance as a function of average space gap of entering vehicles (taken equal for both incoming flows) for some choices of roundabout sizes. In the top graph, exit directions are chosen on an equal basis for incoming cars $P_S = P_E = P_W = P_N = 0.25$ and indicators are assumed to be off. In the bottom graph of Fig. 6, the exit probabilities are chosen on a biased level. In the following graph (Fig. 7), we investigate the dependence of the overall delay on the probability of exit direction in more details. We assume there is a preferential exit direction while the remaining exit probabilities are the same. Total delay is sketched for some choices of the preferred exit direction probabilities. The roundabout circumference is taken to be 24 cells and the in-flows are equal to each other.

We see an interesting result. Under both NS and OV dynamics, the results predicts that over a certain traffic congestion, the overall delay is minimized for the case where the

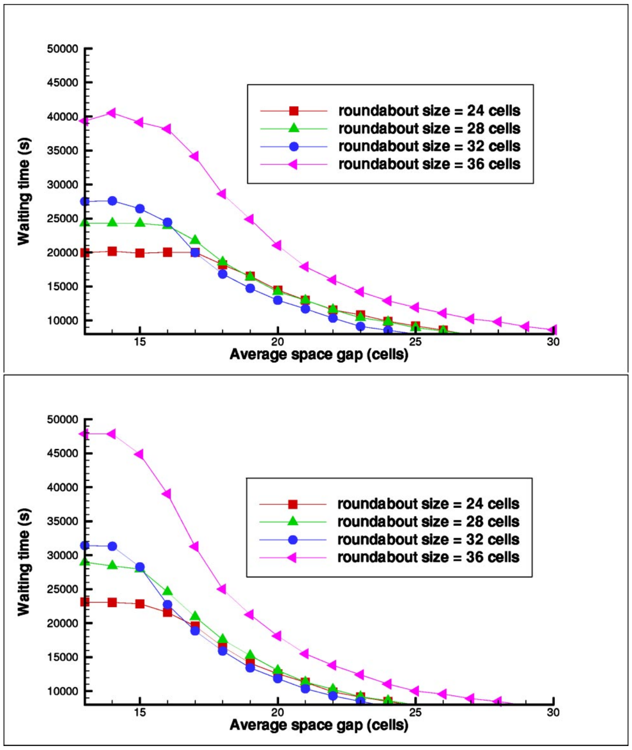

FIG. 6. Overall delay vs average space gap (equal for both streets). Exit probabilities are taken equal for all the exit directions (top). In the bottom graph, the probability of straightforward exit is 0.5 (preferential exit direction) and the probability of exit from the remaining exit directions are equally taken as one-sixth. The graph has been obtained in NS dynamics.

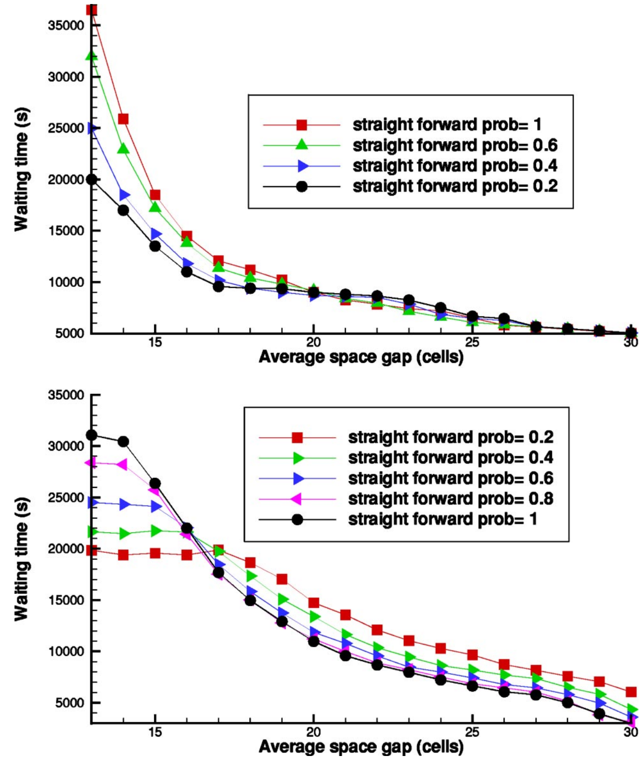

FIG. 7. Overall delay in terms of in-flow for a fixed roundabout size. Probability of forward exiting is varied. The probability of exit from the remaining directions are taken to be equal to each other. The top graph corresponds to OV dynamics. The bottom graph is obtained with NS dynamics.

straightforward exit probability is low. Within NS dynamics, for light traffic states, the lower delay is achieved when the straightforward exit probability is high. This feature is not observed by OV dynamics.

V. COMPARISON TO OTHER CONTROLLING SCHEMES

Let us now compare the roundabout performance with signalized control methods of an intersection. This comparison is our main motive for studying roundabout characteristics. Let us replace the roundabout with an intersection with traffic lights. For simplicity, we consider the intersection of two one-way to one-way streets which are assumed to direct single-lane traffic flow. Basically there are two types of signalization: *Fixed-time* and *traffic adaptive*. We first describe the fixed-time method. In this control scheme, the traffic flow is controlled by a set of traffic lights which are operated on a fixed-cycle. The lights periodically turn green with a fixed period (cycle length) T. This period is divided into two parts: in the first part, the traffic light is green for street A (simultaneously red for street B). This part lasts for T_g seconds ($T_g < T$). In the second part, the lights change color and movement is allowed for the cars of road B. The second part lasts from T_g to T. This behavior is repeated periodically. In [33,34], we have shown that the optimal green time given to street A should be proportional to its in-flow rate. In Fig. 8, we compare the performance of the corresponding roundabout with fixed-time signalization strategy. Traffic volumes are assumed to be equal for both streets. Furthermore, we assume that incoming vehicles cannot turn and should move straight ahead.

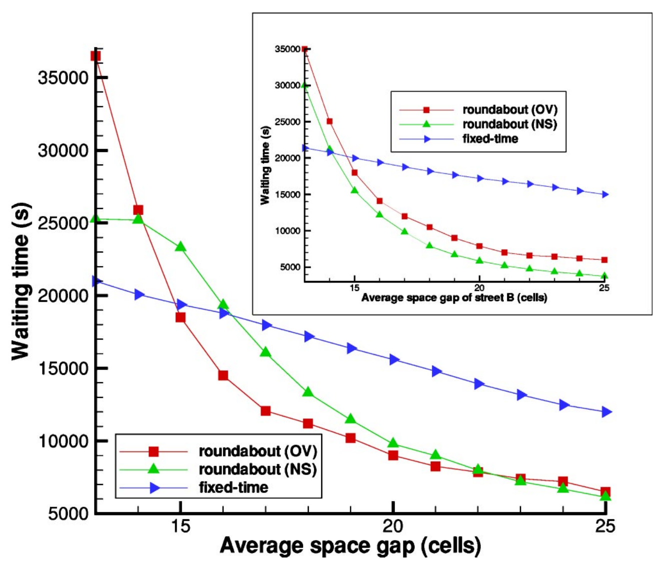

FIG. 8. In signalized controlling, the period of lights is 40 s and the green times are equally distributed to the streets. The inset describes an asymmetric situation in the in-flows. Average space gap of approaching cars are fixed at $\lambda_A = 13$ cells while λ_B is varied. Roundabout size is 24 cells.

According to Fig. 8 in relatively light traffic states, characterized by a large average space gap, a roundabout shows a better performance and gives rise to lower delays. Conversely, in more congested traffic situations, controlling the intersection by signalized traffic light leads to better results. Our simulation results give the critical in-flow rate below which the intersection should be controlled in a self-organized manner. This result can be explained by noting that in sufficiently light traffic states, the approaching cars can easily find the required space gap in the flow of conflicting direction, hence, they can enter the roundabout without spending much times, whereas in a signalized scheme they have to wait at the red parts of the signal even if the flow is negligible in the conflicting direction. This proves that below a certain congestion, the roundabout efficiency is higher than fixed-time signalized. We now discuss the traffic adaptive controlling scheme in which the light signalization is adapted to the traffic at the intersection. Nowadays, advanced traffic control systems anticipate the traffic approaching intersections. These adaptive systems have the capability to dynamically modify the signal timing in response to fluctuating traffic demand. Traffic-responsive methods have shown a very good performance in controlling city traffic, and now a variety of schemes exists in the literature [23,24]. There are various methods for the distribution of green times. In what follows we try to explain some standard ones. In each scheme, the green time of a typical direction is terminated if some conditions are fulfilled:

Scheme (1).

The queue length in the conflicting direction exceeds a cut-off value L_c. This scheme only adapts to the traffic states on the red street.

Scheme (2).

The global car density on the green street falls below the cut-off value ρ_c. Here the algorithm solely adapts to the traffic state in the green street.

Scheme (3).

Each direction is endowed with two control parameters L_c and ρ_c. The green phase is terminated if the conditions: $\rho^g \leqslant \rho_c$ and $L^r \geqslant L_c$ are both satisfied.

In scheme (3), the algorithm implements the traffic states in both streets. The superscripts "*r*" and "*g*" refer to words "red" and "green" respectively. In Fig. 9, we present our simulation results for some types of adaptive signalization schemes introduced above, and compare them to a self-organized scheme by roundabout.

Analogous to the fixed-time method, here we see that below a certain traffic volume, roundabout is more efficient. We note that in the adaptive scheme, the numerical value of critical λ is considerably reduced with respect to fixed-time method. This is due to the advantage of adaptive schemes over fixed-time ones. This comparison has thoroughly been discussed in [34]. Fixed-time predicts (by NS dynamics) $\lambda_c = 16$ cells, while in the adaptive method it goes to $\lambda_c = 21$. Predictions with OV dynamics give quite similar results with λ_c shifted to 15 and 20 cells, respectively.

VI. SUMMARY AND CONCLUDING REMARKS

Traffic signal control is a central issue in the design of advanced traffic management systems. In this regard, the mi-

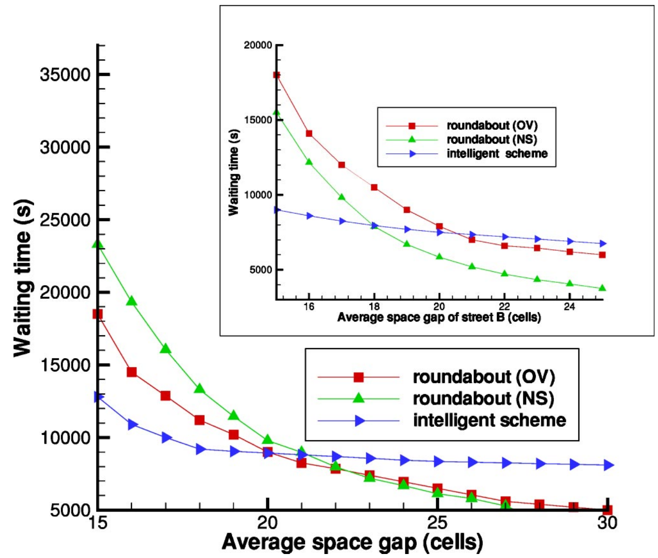

FIG. 9. Overall delay in terms of symmetric average space gap of approaching vehicles. Roundabout size is 24 cells. The critical cutoff length in adaptive strategy is $L_c = 5$. The inset describes an asymmetric situation in the in-flows. Average space gap of approaching cars are fixed at $\lambda_A = 13$ cells while λ_B is varied.

crosimulation of city traffic could be of practical relevance for various applications. Enthusiasm for safety and for the high capacity of roundabouts has resulted in a huge increase in the number of roundabouts. Nevertheless, the efficiency of roundabouts is still under debate and many experts believe that signalized intersections show a better performance in most circumstances. To settle this debate, we have tried to quantitatively explore the basic features of roundabout in order to have better insight into the problem. In this paper, we have investigated the characteristics of traffic at an isolated roundabout in the framework of cellular automata and car-following models. For this purpose, we have developed and analyzed the performance of the various aspects of roundabouts, the most important of which is delay. Our simulation shows that overall delay is significantly affected by roundabout size. Our simulations gives the optimal size for various traffic volumes. The major conclusion shows the existence of critical congestion, dominated by the statistics of arrival space gaps, over which the intersection is made more efficient by signalization strategies. Below this traffic demand, the optimal controlling method is unsignalized roundabout. The critical demand is roughly 550 vehicles/h (corresponding to average gap of 21 cells). Our results shows that the implementation of different dynamical rules for vehicular movement does not change general features of the problem. This supports the idea that, at least in noncongested situations, simple vehicular dynamics are able to simulate the generic aspects. In all of our graphs, we have not observed any substantial difference between the predictions of NS and the optimal velocity models. In the efficient operation of roundabout, the space gap of entering cars are typically 100 m which ensures that the traffic is in free flow phase. However, in this free flow traffic, it is not trivial which signalization schemes acts more optimal. We have attempted to address this question by simulating the vehicular dynamics.

In a more realistic situation, the flow can circulate around the central island via an additional lane. The interior lane should be used by those vehicle intending to make left or U-turns, while the exterior one should be taken by those drivers who tend to turn right or move straightforward. The second interior lane may drastically change the behavior of displayed indicators thus leading to an improvement in the delay. In the present case of single-lane circulations, our simulations imply that the injection of vehicles from more than two entries leads to gridlocking and growing delays. This effect is due to the saturation of circulating flow which hinders the incoming fluxes. The implementation of additional interior lane will certainly remove the blocking and give rise to realistic results. In this general situation, roundabout performance undergoes fundamental changes, and many interesting phenomena arise which we are currently exploring. Finally, we wish to say a few remarks on the role traffic demand fluctuations. Our results have been obtained under the assumption of rather uniform in-flow statistics. However, the flexibility of signalized or the roundabout schemes under strongly fluctuating demand has to be explored in more detail. The work along this line is in progress.

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